

UNIFIED MODEL OF THE RELATIVISTIC πNN AND $\gamma\pi NN$ SYSTEMS

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We present a unified description of the relativistic πNN and $\gamma\pi NN$ systems where the strong interactions are described non-perturbatively by four-dimensional integral equations. Our formulation obeys two and three-body unitarity and is strictly gauge invariant with photons coupled to all possible places in the strong interaction model. At the same time our formulation is free from the overcounting problems plaguing four-dimensional descriptions of πNN and $\gamma\pi NN$ systems. The accurate nature of our description is achieved through the use of the recently introduced gauging of equations method.

I. INTRODUCTION

Recently we have introduced a method for incorporating an external electromagnetic field into any model of hadrons whose strong interactions are described through the solution of integral equations [1,2]. The method involves the gauging of the integral equations themselves, and results in electromagnetic amplitudes where an external photon is coupled to every part of every strong interaction diagram in the model. Current conservation is therefore implemented in the theoretically correct fashion. Initially we applied our gauging procedure to the covariant three-nucleon problem whose strong interactions are described by standard four-dimensional three-body integral equations [1,2]. More recently we used the same method to gauge the three-dimensional spectator equation for a system of three-nucleons [3,4].

Here we apply our gauging procedure to the more complicated case of the covariant πNN system whose four-dimensional integral equations have only recently been derived [5,6]. In this way we obtain gauge invariant expressions for all possible electromagnetic processes of the πNN system, e.g. pion photoproduction $\gamma d \rightarrow \pi NN$, pion electroproduction $ed \rightarrow e'\pi d$, and because pion absorption is taken into account explicitly, we also obtain gauge invariant expressions for processes like deuteron photodisintegration $\gamma d \rightarrow NN$ and Bremsstrahlung $NN \rightarrow \gamma NN$ that are valid even at energies above pion production threshold. Included in the electromagnetic processes described by our model is the especially interesting case of elastic electron-deuteron scattering. Here the deuteron is described as a bound state of our full πNN model; thus, after gauging, we obtain a rich description of the electromagnetic form factors of the deuteron with all possible meson exchange currents taken into account.

II. FOUR-DIMENSIONAL πNN EQUATIONS

Since the early 1960's many attempts have been made to formulate few-body equations for πNN -like systems using relativistic quantum field theory [7–12]. Yet all these attempts have had theoretical inconsistencies with perturbation diagrams being either overcounted or undercounted. The first consistent πNN equations were derived by us only recently [5] and have the feature of containing explicit subtraction terms in the kernels of the equations that eliminate all overcounting. At the same time the undercounting problem is eliminated by the retention of certain three-body forces. The same πNN equations were later derived by Phillips and Afnan [6] using a different method. In this section we would simply like to restate these equations but in a form that is particularly convenient for gauging.

A. Distinguishable Nucleon Case

It is easy to rearrange the four-dimensional πNN equations of Ref. 5 into a convenient form similar to the one used by Afnan and Blankleider [13] in a three-dimensional formulation of the πNN system. For the distinguishable nucleon case we obtain

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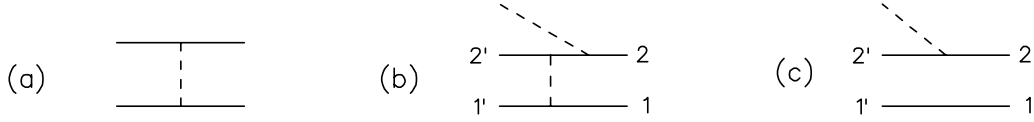


FIG. 1. Illustration of quantities making up V_{NN}^d . (a) V_{NN}^{OPEd} , (b) B , and (c) $F_2 = f_2 d_1^{-1}$.

$$\mathcal{T}^d = \mathcal{V}^d + \mathcal{V}^d \mathcal{G}_t^d \mathcal{T}^d \quad (1)$$

where \mathcal{T}^d , \mathcal{V}^d , and \mathcal{G}_t^d are 4×4 matrices given by

$$\mathcal{T}^d = \begin{pmatrix} T_{NN}^d & \bar{T}_N^d \\ T_N^d & T^d \end{pmatrix}; \quad \mathcal{V}^d = \begin{pmatrix} V_{NN}^d & \bar{\mathcal{F}}^d \\ \mathcal{F}^d & G_0^{-1} \mathcal{I} \end{pmatrix}; \quad \mathcal{G}_t^d = \begin{pmatrix} D_0 & 0 \\ 0 & G_0 w^0 G_0 \end{pmatrix}. \quad (2)$$

Here we use the conventions of Ref. 13 and refer to the two nucleons as particles 1 and 2, and the pion as particle 3; also $\lambda = 1, 2$ labels the channel where nucleon λ and the pion form a two-particle subsystem with the other nucleon being a spectator, and $\lambda = 3$ labels the channel where the two nucleons form a subsystem with the pion being a spectator. The superscript d is used to denote the distinguishable nucleon case on those symbols that will later be used without the superscript for the case of indistinguishable nucleons. Eq. (1) is a symbolic equation representing a Bethe-Salpeter integral equation to be solved for \mathcal{T}^d . \mathcal{T}^d consists of transition amplitudes T_{NN}^d , $T_{\lambda N}$, $T_{N\mu}$, and $T_{\lambda\mu}$ (λ and μ are the spectator-subsystem channel labels) the last three being elements of the matrices T_N^d , \bar{T}_N^d , and T^d , respectively. The physical amplitudes for $NN \rightarrow NN$, $NN \rightarrow \pi d$, $\pi d \rightarrow NN$, and $\pi d \rightarrow \pi d$ are then given by

$$X_{NN}^d = T_{NN}^d; \quad X_{dN}^d = \bar{\psi}_d T_{3N}; \quad X_{Nd}^d = T_{N3} \psi_d; \quad X_{dd}^d = \bar{\psi}_d T_{33} \psi_d, \quad (3)$$

respectively, where ψ_d is the deuteron wave function in the presence of a spectator pion. The kernel \mathcal{V}^d specified in Eq. (2) consists of the following elements:

$$V_{NN}^d = V_{NN}^{OPEd} + \bar{B}^d G_0 B^d - \Delta \quad (4)$$

where V_{NN}^{OPEd} is the nucleon-nucleon one pion exchange potential illustrated in Fig. 1(a), $B^d = B + PBP$ where B is the diagram illustrated in Fig. 1(b) and P is the nucleon exchange operator, and Δ is a subtraction term that eliminates overcounting. \mathcal{F}^d is a 3×1 matrix whose λ 'th row element is given by

$$\mathcal{F}_\lambda^d = \sum_{i=1}^2 \bar{\delta}_{\lambda i} F_i - B^d. \quad (5)$$

Here $\bar{\delta}_{\lambda i} = 1 - \delta_{\lambda i}$ and $F_i = f_i d_j^{-1}$ ($i, j = 1$ or 2 with $i \neq j$) where f_i is the $N_i \rightarrow \pi N_i$ vertex function and d_j is the Feynman propagator of nucleon j . F_2 is illustrated in Fig. 1(c). Note how B^d here plays the role of a subtraction term. $\bar{\mathcal{F}}^d$ is the 1×3 matrix that is the time reversed version of \mathcal{F}^d (similarly for other “barred” quantities), G_0 is the πNN propagator, and \mathcal{I} is the matrix whose (λ, μ) 'th element is $\bar{\delta}_{\lambda, \mu}$. Finally the propagator term \mathcal{G}_t^d is a diagonal matrix consisting of the NN propagator D_0 , and the 3×3 diagonal matrix w^0 whose diagonal elements are $t_1 d_2^{-1}$, $t_2 d_1^{-1}$, and $t_3 d_3^{-1}$, with t_λ being the two-body t matrix in channel λ (for $\lambda = 1$ or 2 , t_λ is defined to be the πN t matrix with the nucleon pole term removed).

The subtraction term Δ is defined with the help of Fig. 2 as

$$\Delta = W_{\pi\pi} + W_{\pi N}^d + W_{NN} + X + Y^d \quad (6)$$

where $W_{\pi N}^d = W_{\pi N} + PW_{\pi N}P$ and $Y^d = Y + PYP$.

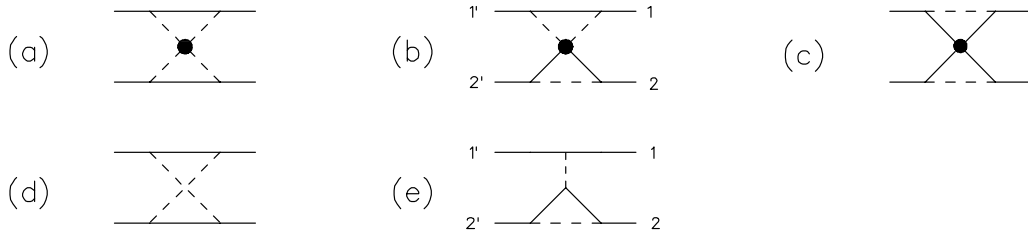


FIG. 2. Terms making up the subtraction term Δ . (a) $W_{\pi\pi}$, (b) $W_{\pi N}$, (c) W_{NN} , (d) X , and (e) Y . The dark circles represent the following two-body amplitudes: (a) full $\pi\pi$ t -matrix, (b) one-nucleon irreducible πN t -matrix, and (c) full NN t -matrix minus the NN one-pion-exchange potential.

B. Indistinguishable Nucleon Case

In approaches based on second quantisation in quantum mechanics it is usual to obtain the scattering equations for identical particles by explicitly symmetrising the equations of the distinguishable particle case. This procedure is not justified in the framework of relativistic quantum field theory. Nevertheless, as we have already derived the πNN equations taking into account identical particle symmetry right from the beginning [5], we can formally deduce how the above distinguishable nucleon πNN equations need to be modified in order to get the indistinguishable nucleon case. With this in mind, we introduce identical nucleon transition amplitudes defined in terms of distinguishable nucleon transition amplitudes as

$$\begin{aligned} T_{NN} &= T_{NN}^d A & T_{\Delta N} &= T_{1N} A & T_{dN} &= T_{3N} A \\ T_{N\Delta} &= \bar{T}_{N1} - \bar{T}_{N2} P & T_{Nd} &= \bar{T}_{N3} A & T_{\Delta\Delta} &= T_{11} - T_{12} P \\ T_{d\Delta} &= T_{31} - T_{32} P & T_{\Delta d} &= T_{13} A & T_{dd} &= T_{33} A \end{aligned} \quad (7)$$

where $A = 1 - P$ is the antisymmetrising operator. As $\bar{T}_{N1} = P\bar{T}_{N2}P$ and $\bar{T}_{N3} = P\bar{T}_{N3}P$, we can alternatively write $T_{N\Delta} = A\bar{T}_{N1}$, $T_{Nd} = A\bar{T}_{N3}$, and $T_{d\Delta} = A\bar{T}_{31}$. Thus in the transition to the indistinguishable particle case, the original 16 transition amplitudes for distinguishable particles have been reduced to 9 antisymmetrised transition amplitudes. By taking residues of the πNN Green function for identical nucleons at two-body subsystem poles, one obtains the following expressions for the physical amplitudes:

$$X_{NN} = T_{NN} ; \quad X_{dN} = \bar{\psi}_d T_{dN} ; \quad X_{Nd} = T_{Nd} \psi_d ; \quad X_{dd} = \bar{\psi}_d T_{dd} \psi_d. \quad (8)$$

Using Eqs. (7) in Eq. (1) it is easy to show that one again obtains a Bethe-Salpeter equation

$$\mathcal{T} = \mathcal{V} + \mathcal{V} \mathcal{G}_t \mathcal{T} \quad (9)$$

but where now \mathcal{T} , \mathcal{V} , and \mathcal{G}_t are 3×3 matrices given by

$$\mathcal{T} = \begin{pmatrix} T_{NN} & T_{N\Delta} & T_{Nd} \\ T_{\Delta N} & T_{\Delta\Delta} & T_{\Delta d} \\ T_{dN} & T_{d\Delta} & T_{dd} \end{pmatrix}; \quad \mathcal{V} = \begin{pmatrix} V_{NN}^d A & A\bar{\mathcal{F}}_1^d & \bar{\mathcal{F}}_3^d A \\ \mathcal{F}_1^d A & -G_0^{-1} P & G_0^{-1} A \\ \mathcal{F}_3^d A & G_0^{-1} A & 0 \end{pmatrix}; \quad (10)$$

$$\mathcal{G}_t = \begin{pmatrix} \frac{1}{2} D_0 & 0 & 0 \\ 0 & G_0 t_1 d_2^{-1} G_0 & 0 \\ 0 & 0 & \frac{1}{4} G_0 t_3 d_3^{-1} G_0 \end{pmatrix} \quad (11)$$

where $t_3 = t_3^d A$ is the t matrix for two identical nucleons in the presence of a spectator pion.

From Eq. (4) it follows that

$$V_{NN}^d A = V_{NN}^{\text{OPE}} - W_{\pi\pi}^R - W_{\pi N}^{LR} - W_{NN}^R - X^R - Y^{LR} + \frac{1}{2} \bar{B}^{LR} G_0 B^{LR} \quad (12)$$

where $V_{NN}^{\text{OPE}} = V_{NN}^{\text{OPE}d} A$ and where we have used a superscript L to indicate that A is acting on the left, and a superscript R to indicate that A is acting on the right. It is easy to see that $W_{\pi\pi}^R = W_{\pi\pi}^L$, $W_{NN}^R = W_{NN}^L$ and $X^R = X^L$.

We note that the πNN equations for identical nucleons derived in Ref. 6, unlike Eqs. (9), (10) and (11), contain no antisymmetrisation operators A and are therefore not equivalent to our πNN equations.

III. πNN ELECTROMAGNETIC TRANSITION CURRENTS

A. Gauging the πNN Equations

In this section we shall derive expressions for the various electromagnetic transition currents of the πNN system. To do this we utilise the recently introduced gauging of equations method [1,2]. As the gauging procedure is identical for the distinguishable and indistinguishable particle cases, we restrict our attention to the πNN system where the nucleons are treated as indistinguishable.

Direct gauging of Eq. (9) gives

$$\mathcal{T}^\mu = \mathcal{V}^\mu + \mathcal{V}^\mu \mathcal{G}_t \mathcal{T} + \mathcal{V} \mathcal{G}_t^\mu \mathcal{T} + \mathcal{V} \mathcal{G}_t \mathcal{T}^\mu \quad (13)$$

which can easily be solved for \mathcal{T}^μ giving

$$\mathcal{T}^\mu = (1 + \mathcal{T}\mathcal{G}_t)\mathcal{V}^\mu(1 + \mathcal{G}_t\mathcal{T}) + \mathcal{T}\mathcal{G}_t^\mu\mathcal{T}. \quad (14)$$

\mathcal{T}^μ is a matrix of gauged transition amplitudes T_{NN}^μ , $T_{N\Delta}^\mu$, T_{Nd}^μ , etc. To obtain the physical electromagnetic transition currents of the πNN system where photons are attached everywhere it is not sufficient to just gauge the physical πNN amplitudes of Eq. (8). Although this would indeed attach photons everywhere inside the strong interaction diagrams, it would miss those contributions to the physical electromagnetic transition currents where the photons are attached to the external (initial and final state) pions and nucleons. In order to also include these external leg contributions it is useful to attach the corresponding propagators to the X -amplitudes of Eq. (8):

$$\tilde{X}_{NN} = D_0 X_{NN} D_0 \quad \tilde{X}_{dN} = d_\pi X_{dN} D_0 \quad (15)$$

$$\tilde{X}_{Nd} = D_0 X_{Nd} d_\pi \quad \tilde{X}_{dd} = d_\pi X_{dd} d_\pi. \quad (16)$$

The physical electromagnetic transition currents are then obtained by gauging Eqs. (15) and (16) and “chopping off” external legs:

$$j_{NN}^\mu = D_0^{-1} \tilde{X}_{NN}^\mu D_0^{-1} \quad j_{dN}^\mu = d_\pi^{-1} \tilde{X}_{dN}^\mu D_0^{-1} \quad (17)$$

$$j_{Nd}^\mu = D_0^{-1} \tilde{X}_{Nd}^\mu d_\pi^{-1} \quad j_{dd}^\mu = d_\pi^{-1} \tilde{X}_{dd}^\mu d_\pi^{-1}. \quad (18)$$

Using Eqs. (8) we obtain that

$$j_{NN}^\mu = D_0^{-1} D_0^\mu T_{NN} + T_{NN} D_0^\mu D_0^{-1} + T_{NN}^\mu \quad (19)$$

$$j_{dN}^\mu = \bar{\phi}_d^\mu D_0 T_{dN} + d_\pi^{-1} \bar{\phi}_d G_0^\mu T_{dN} + \bar{\phi}_d D_0 T_{dN}^\mu + \bar{\phi}_d D_0 T_{dN} D_0^\mu D_0^{-1} \quad (20)$$

$$j_{Nd}^\mu = T_{Nd} D_0 \phi_d^\mu + T_{Nd} G_0^\mu \phi_d d_\pi^{-1} + T_{Nd}^\mu D_0 \phi_d + D_0^{-1} D_0^\mu T_{Nd} D_0 \phi_d \quad (21)$$

$$j_{dd}^\mu = \bar{\phi}_d^\mu D_0 T_{dd} D_0 \phi_d + d_\pi^{-1} \bar{\phi}_d D_0^\mu T_{dd} D_0 \phi_d + \bar{\phi}_d D_0 T_{dd}^\mu D_0 \phi_d + \bar{\phi}_d D_0 T_{dd} D_0^\mu \phi_d d_\pi^{-1} + \bar{\phi}_d D_0 T_{dd} D_0 \phi_d^\mu \quad (22)$$

where ϕ_d is the deuteron bound state vertex function defined by the relation $\psi_d = d_1 d_2 \phi_d$. Note that ϕ_d^μ consists of contributions where the photon is attached everywhere inside the deuteron bound state, and is determined by gauging the two-nucleon bound state equation for ϕ_d [1,2].

That the above equations are gauge invariant is evident from the fact that we have formally attached the photon to all possible places in the strong interaction model. The gauge invariance of our equations also follows from a strict mathematical proof; however, as this proof is essentially identical to the one given for the NNN system [2], we shall not repeat it here.

B. Alternative Form of the πNN Equations

Although the preceding discussion solves the problem of gauging the πNN system in a straightforward way, the expression obtained to calculate the gauged transition amplitudes, Eq. (14), is not the most convenient for numerical calculations. The disadvantage of Eq. (14) is that it utilises a Green function \mathcal{G}_t which contains two-body t matrices. The presence of these t matrices makes the evaluation of \mathcal{T}^μ unnecessarily complicated. We therefore present an alternative form of the πNN equations which uses a “free” Green function which contains no two-body interactions and which leads to simpler expressions for the πNN electromagnetic transition currents.

The πNN equations Eqs. (9), (10) and (11), can be written in the form

$$\begin{pmatrix} T_{NN} & \bar{T}_N \\ T_N & T \end{pmatrix} = \begin{pmatrix} V_{NN} & \bar{\mathcal{F}} \\ \mathcal{F} & LG_0^{-1} \end{pmatrix} \left[1 + \begin{pmatrix} \frac{1}{2}D_0 & 0 \\ 0 & G_0 t G_0 \end{pmatrix} \begin{pmatrix} T_{NN} & \bar{T}_N \\ T_N & T \end{pmatrix} \right]. \quad (23)$$

where $V_{NN} = V_{NN}^d A$ is given by Eq. (12), and

$$\mathcal{F} = \begin{pmatrix} \mathcal{F}_1^d A \\ \mathcal{F}_3^d A \end{pmatrix} = \begin{bmatrix} (F_2 - B^d)A \\ (F_1 + F_2 - B^d)A \end{bmatrix}, \quad (24)$$

$$\bar{\mathcal{F}} = (A\bar{\mathcal{F}}_1^d \ A\bar{\mathcal{F}}_3^d) = [A(\bar{F}_2 - \bar{B}^d) \ A(\bar{F}_1 + \bar{F}_2 - \bar{B}^d)], \quad (25)$$

$$L = \begin{pmatrix} -P & A \\ A & 0 \end{pmatrix}, \quad t = \begin{pmatrix} t_1 d_2^{-1} & 0 \\ 0 & \frac{1}{4} t_3 d_3^{-1} \end{pmatrix}. \quad (26)$$

With the view of gauging Green function versions of πNN transition amplitudes, we introduce the Green function matrix appropriate to Eq. (23):

$$\begin{aligned} \mathcal{G} &= \begin{pmatrix} G_{NN} & \bar{G}_N \\ G_N & G \end{pmatrix} \equiv \begin{pmatrix} G_{NN} & G_{N\Delta} & G_{Nd} \\ G_{\Delta N} & G_{\Delta\Delta} & G_{\Delta d} \\ G_{dN} & G_{d\Delta} & G_{dd} \end{pmatrix} \\ &= \begin{pmatrix} AD_0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} D_0 & 0 \\ 0 & G_0 \end{pmatrix} \begin{pmatrix} T_{NN} & \bar{T}_N \\ T_N & T \end{pmatrix} \begin{pmatrix} D_0 & 0 \\ 0 & G_0 \end{pmatrix}. \end{aligned} \quad (27)$$

The inhomogeneous term is chosen so that G_{NN} corresponds exactly to the full Green function for NN scattering. Then it can be shown that \mathcal{G} satisfies the equation

$$\mathcal{G} = \begin{pmatrix} AD_0 & 0 \\ 0 & LG_0 \end{pmatrix} \left[1 + \begin{pmatrix} \frac{1}{4} V_{NN} - \frac{1}{4} \bar{\Lambda} L G_0 \Lambda & \frac{1}{2} \bar{\Lambda} \\ \frac{1}{2} \Lambda & t \end{pmatrix} \mathcal{G} \right] \quad (28)$$

where Λ and $\bar{\Lambda}$ are given by

$$\Lambda = \begin{bmatrix} (F_1 - \frac{1}{2} B^d) A \\ -\frac{1}{2} B^d \end{bmatrix}; \quad \bar{\Lambda} = [A(\bar{F}_1 - \frac{1}{2} \bar{B}^d) \quad -\frac{1}{2} \bar{B}^d]. \quad (29)$$

The essential feature of Eq. (28) is that it is written in terms of an effective “free” Green function matrix

$$\mathcal{G}_0 = \begin{pmatrix} AD_0 & 0 \\ 0 & LG_0 \end{pmatrix} \quad (30)$$

which does not involve two-body interactions. For this reason Eq. (28) is ideal for the purposes of gauging.

C. Gauging the Alternative Form of the πNN Equations

In terms of the elements of \mathcal{G} , the Green function versions of the physical amplitudes (defined in Eq. (16)) are given by

$$\tilde{X}_{NN} = G_{NN} \ ; \quad \tilde{X}_{dN} = \bar{\phi}_d G_{dN} \ ; \quad \tilde{X}_{Nd} = G_{Nd} \phi_d \ ; \quad \tilde{X}_{dd} = \bar{\phi}_d G_{dd} \phi_d. \quad (31)$$

After gauging, these equations give

$$\tilde{X}_{NN}^\mu = G_{NN}^\mu \quad \tilde{X}_{dN}^\mu = \bar{\phi}_d^\mu G_{dN} + \bar{\phi}_d G_{dN}^\mu \quad (32)$$

$$\tilde{X}_{Nd}^\mu = G_{Nd}^\mu \phi_d + G_{Nd} \phi_d^\mu \quad \tilde{X}_{dd}^\mu = \bar{\phi}_d^\mu G_{dd} \phi_d + \bar{\phi}_d G_{dd}^\mu \phi_d + \bar{\phi}_d G_{dd} \phi_d^\mu. \quad (33)$$

The πNN electromagnetic transition currents $j_{\alpha\beta}^\mu$ are then determined by Eqs. (17) and Eqs. (18).

To determine the quantities $G_{\alpha\beta}^\mu$ in Eqs. (32) and (33) we need to derive the expression for \mathcal{G}^μ by gauging Eq. (28). Defining

$$\mathcal{V}_t = \begin{pmatrix} \frac{1}{4} V_{NN} - \frac{1}{4} \bar{\Lambda} L G_0 \Lambda & \frac{1}{2} \bar{\Lambda} \\ \frac{1}{2} \Lambda & t \end{pmatrix} \quad (34)$$

Eq. (28) can be written as

$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0 \mathcal{V}_t \mathcal{G}. \quad (35)$$

Gauging this equation and solving for \mathcal{G}^μ gives

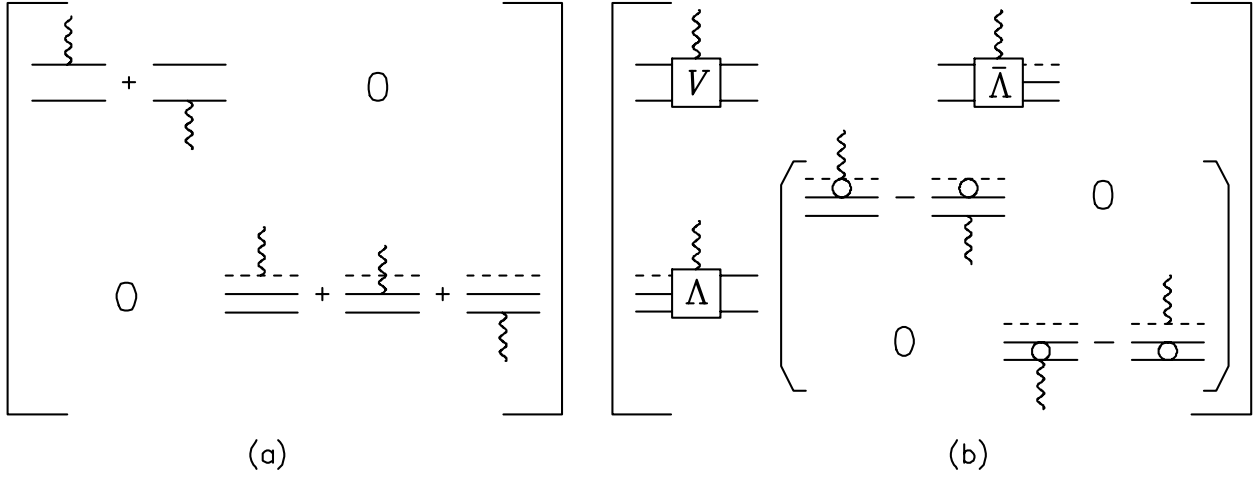


FIG. 3. Graphical representation of (a) \mathcal{M}^μ and (b) \mathcal{V}_t^μ which enter into the expression for the gauged Green function matrix \mathcal{G}^μ of Eq. (40). Constant factors and effects of operator A and matrix Ω have been ignored in this illustration.

$$\mathcal{G}^\mu = (1 + \mathcal{G}\mathcal{V}_t)\mathcal{G}_0^\mu(1 + \mathcal{V}_t\mathcal{G}) + \mathcal{G}\mathcal{V}_t^\mu\mathcal{G}. \quad (36)$$

To simplify this equation we cannot use Eq. (35) to write $1 + \mathcal{V}_t\mathcal{G} = \mathcal{G}_0^{-1}\mathcal{G}$ and $1 + \mathcal{G}\mathcal{V}_t = \mathcal{G}\mathcal{G}_0^{-1}$ since L is singular so that the inverse \mathcal{G}_0^{-1} does not exist. Instead we use the fact that $L = L\Omega L$, where

$$\Omega = \begin{pmatrix} -\frac{1+P}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{4} \end{pmatrix} \quad (37)$$

which allows us to write

$$\mathcal{G}_0^\mu = \mathcal{G}_0\mathcal{M}^\mu\mathcal{G}_0 \quad (38)$$

where

$$\mathcal{M}^\mu = \begin{pmatrix} \frac{1}{2}AD_0^{-1}D_0^\mu D_0^{-1} & 0 \\ 0 & \Omega G_0^{-1}G_0^\mu G_0^{-1} \end{pmatrix}. \quad (39)$$

Using Eq. (38) in Eq. (36) gives us a compact expression for \mathcal{G}^μ :

$$\mathcal{G}^\mu = \mathcal{G}(\mathcal{M}^\mu + \mathcal{V}_t^\mu)\mathcal{G}. \quad (40)$$

It is easy to see that the term \mathcal{M}^μ corresponds to photon coupling in the impulse approximation while \mathcal{V}_t^μ corresponds to the interaction currents and consists of the elements $V^\mu = (V_{NN} - \bar{\Lambda}LG_0\Lambda)^\mu$, $\bar{\Lambda}^\mu$, Λ^μ and t^μ . It is important to note that the diagonal elements of matrix t^μ are both of the form

$$(t_i d_j^{-1})^\mu = t_i^\mu d_j^{-1} + t_i (d_j^{-1})^\mu = t_i^\mu d_j^{-1} - t_i \Gamma_j^\mu \quad (41)$$

where $\Gamma_i^\mu = d_i^{-1}d_i^\mu d_i^{-1}$ is the electromagnetic vertex function of the nucleon and the last equality follows from the fact that $(d_j^{-1}d_j)^\mu = 0$. Thus the diagonal elements of t^μ involve new subtraction terms $t_i \Gamma_j^\mu$ whose origin does not lie in the subtraction terms of the strong interaction πNN equations, but rather in the gauging procedure itself. Analogous subtraction terms arise in the three-nucleon problem whose strong interaction equations have no subtraction terms [1,2]. Similar subtraction terms will arise in the gauging of F_1 and \bar{F}_1 . The graphical representation of \mathcal{M}^μ and \mathcal{V}_t^μ is given in Fig. 3.

Eq. (40) can be used to determine all the possible electromagnetic transition currents of the πNN system. An especially interesting use of Eq. (40) is to study the electromagnetic properties of bound states of the πNN system. It is certainly expected that the strong interaction πNN model under discussion admits a bound state corresponding to the physical deuteron. In this case a solution will exist to the homogeneous version of Eq. (9):

$$\Phi = \mathcal{V}\mathcal{G}_t\Phi \quad (42)$$

where $\Phi = (\Phi_N \ \Phi_\Delta \ \Phi_d)^T$. Here Φ_N is the usual deuteron vertex function describing the $d \rightarrow NN$ transition, while Φ_Δ and Φ_d are somewhat unusual in that they describe transitions to clustered πNN states: $d \rightarrow (\pi N)N$ and $d \rightarrow (NN)\pi$ respectively. Comparing Eq. (27) and Eq. (28) it is seen that \mathcal{G} has a pole at the deuteron mass M_d :

$$\mathcal{G} \sim i \frac{\Psi \bar{\Psi}}{P^2 - M_d^2} \quad \text{as} \quad P^2 \rightarrow M_d^2 \quad (43)$$

where P is the total four-momentum of the system, and where Ψ satisfies the equation

$$\Psi = \mathcal{G}_0 \mathcal{V}_t \Psi. \quad (44)$$

Clearly Ψ is related to Φ by the equation $\Psi = \begin{pmatrix} D_0 & 0 \\ 0 & G_0 \end{pmatrix} \Phi$ so that either of the equations (42) or (44) can be used to determine Ψ . Taking the left and right residues at the deuteron bound state poles of Eq. (40) we obtain the bound state electromagnetic current

$$j^\mu = \bar{\Psi} (\mathcal{M}^\mu + \mathcal{V}_t^\mu) \Psi \quad (45)$$

which describes the electromagnetic properties of the deuteron whose internal structure is described by the present πNN model. Eq. (45) provides a very rich description of the internal electromagnetic structure of the deuteron with all possible meson exchange currents being taken into account in a gauge invariant way. In view of the accuracy of this model which is based on meson and baryon degrees of freedom, a comparison of the deuteron electromagnetic form factors (easily extracted from j^μ) with experiment should prove to be very interesting.

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